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Discrete Optimization

The Transit Route Arc-Node Service Maximization problem

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ABSTRACT

This article presents a new method for determining optimal transit routes. The Transit Route Arc-Node Service Maximization model is a mathematical model that maximizes the service value of a route, rather than minimizing cost. Cost (distance) is considered as a budget constraint on the extent of the route. The mathematical formulation modifies and exploits the structure of linear programming problems designed for the traveling salesman problem. An innovative divide-and-conquer solution procedure is presented that not only makes the transit routing problem tractable, but also provides a range of high-quality alternate routes for consideration, some of which have substantially varying geometries. Variant formulations are provided for several common transit route types. The model is tested through its application to an existing street network in Richardson, TX. Optimal numeric results are obtained for several problem instances, and these results demonstrate that increased route cost is not correlated with increased service provision.

1. Introduction

The primary objective of this article is to outline a new method for determining optimal transit routes, such that a service value for the route is maximized. Although transit planners may enjoy the prospect of developing a set of entirely new routes for a complete transit system based on some notion of optimality, this is not feasible for transit agencies that have been in operation for some time. Those transit agencies that have provided service along established routes must respect the existing travel patterns that they helped create among the population being currently served. This suggests that incremental changes in route structure could be implemented, and that these minor changes should reflect changes in the service values along the route. Such changes can take the form of modifications that create train feeder routes (Quadrifoglio and Li, 2009; Shrivastava and O'Mahony, 2006, 2009; Verma and Dhingra, 2005) or urban circulator routes (Cornillie, 2008; Lownes and Machemehl, 2008, 2010), or modifications to serve newly developed areas with route extensions (Matisziw et al., 2006). Moreover, due to monetary resource constraints, transit agencies can be forced to reduce the cost of a route while still providing service to that population (Zhou et al., 2008). In this case, the agency does not want the minimum cost route, but rather the route that will provide the maximum service to the population given the cost constraints imposed on them. Service orientation has been described as one of the most important decisions that transit managers can make (Brown and Thompson, 2008).

In order to address this issue, this article presents the Transit Route Arc-Node Service Maximization (TRANSMax) model. TRANSMax is a mathematical model that maximizes the overall service value of a route rather than attempting to minimize cost. Cost or distance is considered as a constraint on the extent of the route. The service value of a route is a function of the service values on nodes which are intersections of the street network, and the service values of the arcs connecting those nodes. Service values can be a function of the population, employment opportunities, and other measures of route attractiveness given their access to the bus route. The mathematical formulation of the TRANSMax model borrows from the structure of linear programming problems designed for both the traveling salesman problem and the vehicle routing problem. A solution procedure designed to exploit this formulation is outlined. This method serves to not only make the transit routing problem tractable by dividing the problem into smaller sub-problems, but it also provides a range of high-quality alternate routes for consideration in the decision making process.

In the following section the literature regarding transit route determination is reviewed, with particular focus on the objectives to be optimized, whether or not single or multiple route systems are determined, the range of solution procedures for optimal transit route problems, and the rationale for service maximization. The TRANSMax formulation is then presented with an explanation of its components followed by a description of a three stage solution procedure. Computational experience with the TRANSMax model is gained through its application to an existing street network in

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Richardson, TX. Optimal numeric results are obtained for several problem instances.

2. Literature review

2.1. Cost minimization and multiple objectives

From their earliest incarnations vehicle routing problems (VRPs) have been formulated as distance or cost minimization problems (Balas, 1989; Clarke and Wright, 1964; Dantzig and Ramser, 1959; Flood, 1956; Kulkarni and Bhave, 1985). This overwhelming bias has persisted, as demonstrated by a review article (Chien and Yang, 2000) where nine out of ten research articles regarding transit route design written between 1967 and 1998 employed a total cost-minimization objective. The transit cost is nearly always formulated as a generalized measure of operator costs (List, 1990), user costs (Dubois et al., 1979; Silman et al., 1974), or both operator and user costs (Ceder, 2001; Ceder and Israeli, 1998; Ceder and Wilson, 1986; Chien and Yang, 2000; Chien et al., 2001; Chien et al., 2003; Chien and Qin, 2004; Lampkin and Saalmans, 1967; Mauttone and Urquhart, 2009; Newell, 1979; Wang and Po, 2001; Zhao and Zeng, 2007). The few exceptions include a model that maximizes consumer surplus (Hasselström, 1981), a model that seeks to maximize the number of public transport passengers (Van Nes et al., 1988), a model that seeks equity among users (Bowerman et al., 1995), and models that seek to minimize transfers while encouraging route directness and demand coverage (Zhao et al., 2005; Zhao and Gan, 2003).

A substantial subset of the literature posits that the transportation network planning problem is one that is not captured well by any single optimization objective, but rather multiple objectives should be considered (Current and Marsh, 1993; Current and Min, 1986; Jozefowiez et al., 2008). Among the proposed multi-objective models are those that tradeoff maximal covering of demand against minimizing distance (or cost) for single routes (Current and Pirkul, 1994; Current et al., 1984a,b, 1985; Current and Schilling, 1989) and multiple routes (Wu and Murray, 2005), those that seek to both minimize cost and maximize accessibility in terms of distance traveled (Current et al., 1987; Current and Schilling, 1994), and those that tradeoff access with service efficiency (Murray, 2003; Murray and Wu, 2003). These multi-objective approaches to transit route design are often said to optimize total welfare (Kepaptsoglou and Karlaftis, 2009). Clearly, multi-objective approaches to transit route planning allow for a more comprehensive examination of factors that contribute to transit use and operations. However, the present research focuses on the particular problem of maximizing service values for transit routes in the presence of a cost constraint. It is known from experience that this problem occurs frequently in transit planning operations, generally as a result of new budgets imposed on planners.

2.2. Individual vs. system route design and solution procedures

Although much valuable research has been conducted on the problem of determining an optimal or near-optimal set, or network, of routes for an entire transit system, (Ceder and Wilson, 1986; Chakroborty and Dwivedi, 2002; List, 1990; Silman et al., 1974), many opportunities exist for making incremental changes to transit routes when conditions change. It has been shown that individual bus routes can be improved within a set of political and economic constraints (Matisziw et al., 2006), that relocating bus routes can reduce operating cost or improve route accessibility (Chien et al., 2001), and that these design choices can influence the effectiveness of strategic long-term planning and capital investment decisions (Magnanti and Wong, 1984). Even small changes in bus operations have been shown to have a dramatic influence on bus system performance (Fernandez and Tyler, 2005). Perhaps most importantly, the changes in service provision due to bus route changes brought on by fiscal constraints or other planning decisions can elicit strong reactions from the populations being served and their political representatives (Clark, 2009; Dresser, 2005; Sutton, 2009). This suggests that an objective method for determining the route that maximizes service under such constraints would serve as positive tool for dialog during the planning process.

Given the combinatorial complexity of transit system development problems, it may be impractical or impossible to obtain optimal solutions for large problem instances. When this is the case, alternate—though not guaranteed optimal—solution procedures can be employed. These include formal heuristic (or approximate) methods to quickly find good transit routes (Bowerman et al., 1995; Chien and Yang, 2000; Fan and Machemehl, 2008; Fan and Machemehl, 2004; Mauttone and Urquhart, 2009; Van Nes et al., 1988; Zhao and Gan, 2003), heuristics based on genetic algorithms (Chien et al., 2001; Tom and Mohan, 2003; Agrawal and Mathew, 2004) or other procedures with a stochastic element (Fan and Machemehl, 2006; Yang et al., 2007; Zhao and Zeng, 2006), heuristics that incorporate expert user input in the process (Baaj and Mahmassani, 1995; Ceder and Wilson, 1986; Lampkin and Saalmans, 1967; Shih et al., 1998), and heuristics that are entirely based on expert user experience. The last of these is the most widely used and perhaps the most important technique for the majority of transit agencies and is sometimes referred to as manual route planning (Dubois et al., 1979; Moorthy, 1997). With this method a transit planner uses his or her knowledge of the area to be served and their intuitive understanding of the entire route planning process in order to generate good route alternatives. Subsequent changes to routes are determined by driving through the area and noting additional service opportunities (e.g. new apartment complexes, new employment or shopping opportunities) on a map. The authors concur with those who find that the expertise of transit planners employing the manual method can result in very good solutions and that the ability of such experts to quickly react to customer demands and complaints is a valuable asset (Newell, 1979). However, the solutions determined heuristically in this way are likely to be suboptimal and without an optimal solution process there is no way to determine the extent to which the manual solutions are suboptimal. Lastly, there has recently been increased interest in the use of simulation methods, such as agent based modeling and cellular automaton models to replicate bus behavior (Jiang et al., 2003).

According to the existing literature it has been suggested that solutions for mathematical programming approaches to transit route design are inevitably heuristic due to the combinatorial complexity of the problems (Ceder and Israeli, 1998; Chien et al., 2001; Dubois et al., 1979; Lampkin and Saalmans, 1967; Shih et al., 1998). While this may still be true for the determination of a system of routes, the present research shows that an approach based on service maximization rather than cost minimization, that determines a single route in a limited area with a cost constraint, allows for guaranteed optimal solution procedures to be employed.

2.3. Service maximization

This research asserts that minimization of cost is not an appropriate measure of bus route optimality for two reasons. First, in the absence of an additional constraint on a minimum acceptable level of demand served, the objective of minimizing cost could lead to very low cost routes that serve little if any demand. Secondly, a cost-minimization objective presumes that underperforming bus routes could be eliminated, when in fact there may be social or political pressure to keep some variation of those routes in operation. In practice the primary objective of the operator is not to reduce costs, but to provide as much service as possible, as efficiently as possible, while operating within cost constraints. It has been noted that the provision of service is the only tangible product perceived by users (Norman, 2003), and that strategic access provision is an important element in the ongoing regional transportation planning process (Murray, 2001; Murray et al., 1998).

The service values to be maximized can represent several characteristics of the transit area. Service values could be a function of the population or population density surrounding the network in residential areas. There are a number of methods for determining the population with access to a transit route including area based (Hsiao et al., 1997), line-based (O'Neill et al., 1992), and point based (Biba et al., 2010). Similarly, the service values could be based on the number and size of establishments in commercial areas, or the number of employees in industrial or manufacturing centers (Modarres, 2003). From a modeling perspective, service values could be associated with nodes in the transport network, with the arcs connecting those nodes, or with both. Arc values are important in problems such as mail delivery or snow removal, while node values are considered important in most pick-up and delivery problems. One study found that aggregation of customers from arcs to nodes led to improvements in solution times and solution quality (Oppen and Lokketangen, 2006). Research has shown that such representational choices can have a significant influence on research outcomes (Horner and Murray, 2004) and thus should be made only after careful consideration of the modeling environment and the operational objectives. The TRANSMax model described below allows for both node and arc representations of demand for service to be employed at the user's discretion.

Service values can change over time as land uses change. The construction of a new large apartment complex would increase the potential service on its associated segments of the network. Service dynamics of the network can change when stores go out of business or when employers open new facilities or expand or contract operations at existing facilities. These changes create an opportunity to re-evaluate the optimal transit route through that area.

3. TRANSMax integer programming formulation

The problem to be solved is that of finding an optimal transit route. If operating costs are seen as a constraint on the optimal route, an appropriate objective is to maximize service value. When service values are associated with both arcs and nodes in the network the problem becomes the Transit Route Arc-Node Service Maximization (TRANSMax) problem. This problem determines the optimal combination of connected arcs and nodes that will constitute the route that best serves the transit area. In this section a mathematical formulation of the TRANSMax problem is presented and discussed. This is followed by a discussion of a series of variant constraint formulations.

Perhaps the most well known vehicle routing problem is the Traveling Salesman Problem (TSP). Unfortunately, formulations for the TSP are not sufficient by themselves to address the TRANSMax problem. The TSP seeks the best route through a known set of cities, while the stops on an optimal transit route are not known in advance. However, some constructions designed for the TSP can be adapted to the transit route problem Flood (1956) proposed a set of subtour elimination constraints for the (TSP) that perform similar duty to conservation of flow constraints in the context of maximal flow problems. Using the notation of Vajda (1961) these constraints take the form:

$$\sum_{i=1}^{m} x_{ijt} - \sum_{i=1}^{m} x_{ji(t+1)} = 0 \quad \text{for } j = 1, 2, \ldots, m; \; t = 1, 2, \ldots, m, \tag{1}$$

where m is the number of cities to be visited, i and j are indices of cities, and t represents the sequence of arcs in the route. Thus x_{ijt} is the binary decision variable associated with the tth arc in the route which goes from city i to city j. These constraints require that any arc entering j on the tth step of the route must have a corresponding arc exiting j on the subsequent ($t + 1$) step of the route (or at time step 1 if $t = m$). This guarantees a connected route, and therefore eliminates disconnected subtours. Other constraints in the well-known TSP formulation require that exactly one step of the route enter each city, thus only one arc exits each city. While variations of these constructions have been incorporated into the TRANSMax formulation, there are several fundamental differences from the TSP. The objective is to maximize service provision rather than minimize cost, and the number of stops—and arcs connecting those stops—are not known in advance. This latter characteristic demands an additional step in the solution procedure (discussed in detail below) to determine the best number of stops to include in the solution.

The general TRANSMax formulation consists of:

$$\text{Maximize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{r=1}^{R} (A_{ij} + N_i) x_{ijr} \tag{2}$$

$$\text{Subject to}: \sum_{i=1}^{m} \sum_{r=1}^{R} x_{ijr} \leqslant 1 \quad \text{for } j = 1, 2, \ldots, m, \tag{3}$$

$$\sum_{j=1}^{m} \sum_{r=1}^{R} x_{ijr} \leqslant 1 \quad \text{for } i = 1, 2, \ldots, m, \tag{4}$$

$$\sum_{i=1}^{m} x_{ijr} - \sum_{i=1}^{m} x_{ji(r+1)} = 0 \quad \text{for } j = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, R-1, \tag{5}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{m} x_{ijr} = 1 \quad \text{for } r = 1, 2, \ldots, R, \tag{6}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{r=1}^{R} d_{ij} x_{ijr} \leqslant D, \tag{7}$$

where: i and j are indices of nodes; m is the number of nodes in the network; r is the index of arcs comprising a route; R is the maximum number of arcs in a route; A_{ij} is the service value associated with the arc from node i to node j; N_i is the service value associated with node i; d_{ij} is the length of the arc between node i and node j; D is the maximum length of the route; and x_{ijr} is a decision variable equal to 1 if the arc from i to j is chosen for step r in the route, and 0 otherwise.

The objective function (2) seeks to maximize the sum of the arc and node service values for a route, which is comprised of R arcs. In order to ensure that a complete, connected, non-overlapping route is generated, constraints (3) require that no more than one arc entering any node be selected, and constraints (4) require that no more than one arc exiting a node be selected. Together, constraints (3) and (4) eliminate the possibility that the route will cross over itself, in effect serving the same demand twice, and eliminate the possibility that the route will backtrack over itself since U-turns are generally not permitted for busses. These inequalities differ from equality constraints in TSP formulations since the optimal transit route will not necessarily visit every node in the network. Constraints (5) require that if an arc enters a node on step r of the route, an arc exiting that node must be chosen for step $r + 1$ of the route. Constraints (6) ensure that there will be exactly one arc chosen for any step in the route. Constraint (7) represents the distance, cost, or time constraint on the route. The TRANSMax Model is combinatorially complex. There are ($m^{2*}R$) variables, and $2m + mR + R + 1$ constraints in each problem instance. Moreover, as will be shown below many problem instances may need to be solved for a range of values for R.

The general TRANSMax formulation can be modified based on additional constraints on the route geometry. For example, it may be that the route must begin and end at the same point, creating a loop. A generic loop constraint where the starting and ending points are the same, but unspecified, can be formulated in the following way:

$$\sum_{j=1}^{m} x_{ij1} - \sum_{j=1}^{m} x_{jiR} = 0 \quad \text{for } i = 1, 2, \ldots, m. \tag{8}$$

Constraints (8) demand that for each node i either an arc leaving the node on step one and an arc entering the node on the final step of the route will be chosen, or node i will not be the starting and ending point of the route. Node i is still eligible to be included at some other step of the route. Constraints (3) and (4) continue to limit the number of arcs entering and exiting each node including the starting/ending point of the loop.

Additional constraints may be added if a particular point of interest must be included in the route due to assets or facilities at that location. As an example it is common to have circulator bus routes that begin and end at transit centers or rail stations. This variation of a loop constraint can be formulated in the following way:

$$\sum_{j=1}^{m} x_{sj1} = 1, \tag{9}$$

$$\sum_{i=1}^{m} x_{isR} = 1, \tag{10}$$

where s represents the node at which the station is located and R is the number of arcs in the route. Constraint (9) ensures that the arc that represents the first step of the route must begin at the station. Constraint (10) ensures that the last step of the route re-enters the station node. Together these constraints force the route to be a closed loop beginning and ending at the station.

If a route is being designed to collect people from the service area and deliver them to a single fixed point, this is termed a feeder route. In this case only constraint (9) or constraint (10) should be implemented. Choosing either will give the same optimal result. This results in a chain of arcs that is fixed at one end on the station and extends to some unspecified point in the service area in such a way that the service value is maximized without violating the distance or cost constraint.

Similarly, variants of these constraints can constrain the route to transfers riders from one station to another. This can be accomplished using constraint (9) as it appears above, with a small modification to constraint (10) including a new ending station, e:

$$\sum_{i=1}^{m} x_{ieR} = 1. \tag{11}$$

In any of the variant formulations given above, it may be that significant points of interest must be visited along the route, but they do not necessarily act as stations or terminal points. These waypoints may be large employment or commercial centers that are considered desirable points to be served by the route. Waypoint constraints can take the following form:

$$\sum_{j=1}^{m} \sum_{r=1}^{R} x_{wjr} + \sum_{i=1}^{m} \sum_{r=1}^{R} x_{iwr} \geqslant 2, \tag{12}$$

where w is the location of the desired waypoint.

Considered as a whole, Eqs. (2)–(12) provide for a family of formulations that allow optimal routes to be defined under a variety of circumstances. The variant constraints can be combined to produce a diverse assortment of route types (see Fig. 2). Given this flexible framework the challenge remains to demonstrate that optimal solutions to practical instances of the TRANSMax problem can be achieved.

4. Solution methodology

Solving the TRANSMax model involves a three stage analytical procedure. The first stage uses a network reduction heuristic to eliminate network features that cannot logically be included in the optimal solution to the routing problem. Since the optimal number of arcs in the route (the best value for R) is not known in advance, the second stage entails determining the range of possible values for the number of arcs in the optimal route. The third and final stage consists of solving instances of the TRANSMax model for all values of R in that possible range in order to exhaustively determine the global optimal solution, while additionally providing a range of near-optimal solutions. Each of these three stages is explained in more detail below.

Given the combinatorial complexity of network routing problems and the desire to find optimal solutions, the smallest possible network should be used for analysis. A reduction in the number of nodes and arcs in turn reduces the number of constraints and variables in the linear programming formulation, which is likely to reduce the solution time. Consider an instance of the TRANSMax problem where the goal is to determine the optimal route that both begins and ends at a central train station. This loop route is not permitted to cross itself or retrace itself. Based on these goals and constraints, a subset of the arcs in the transit network can be logically eliminated from consideration for routes, thus reducing the size of the problem. In this instance, this is done by evaluating two shortest paths for each node in the network. The first shortest path is simply the shortest path from the station to the node under examination. The second shortest path is the shortest path back from the node to the station that does not employ any arcs or nodes that were in the first shortest path. The sum of these paths is the length of the shortest possible loop from (and to) the station that passes through that node (and meets the other route constraints). If the length of that loop is greater than the distance constraint value in the model (D in constraint (7)), that node could not logically be part of the optimal loop route, and therefore its elimination could not possibly influence the optimal route solution procedure to follow. When that is the case, both the node itself, and any arcs incident to it, can be eliminated from the network under consideration. Fig. 1 shows the entire network in light grey, with the subset of arcs that remain under consideration (darker grey) after the heuristic is applied. While this process is heuristic in the sense that it may be possible to generate methods that even further reduce the size of the network prior to optimization, it should be noted again that this procedure eliminates only network arcs that could not possibly influence the optimal route determination to follow, and thus does not influence the capability of the entire three stage solution process to achieve global optimality.

Other route types (linear or radial) and their associated constraint sets demand variations on this network reduction heuristic. For simple linear routes extending from a station a single shortest path could be applied. For routes between known points, a k-shortest path algorithm that is permitted to run until paths are found that exceed the distance constraint could be implemented. It should be noted that, while these methods employ well-known and efficient shortest path algorithms, the extent to which the network will be pruned by these methods will be a function of the type of optimal route to be determined, the distance constraint value, and the nature of the network itself. Moreover, the

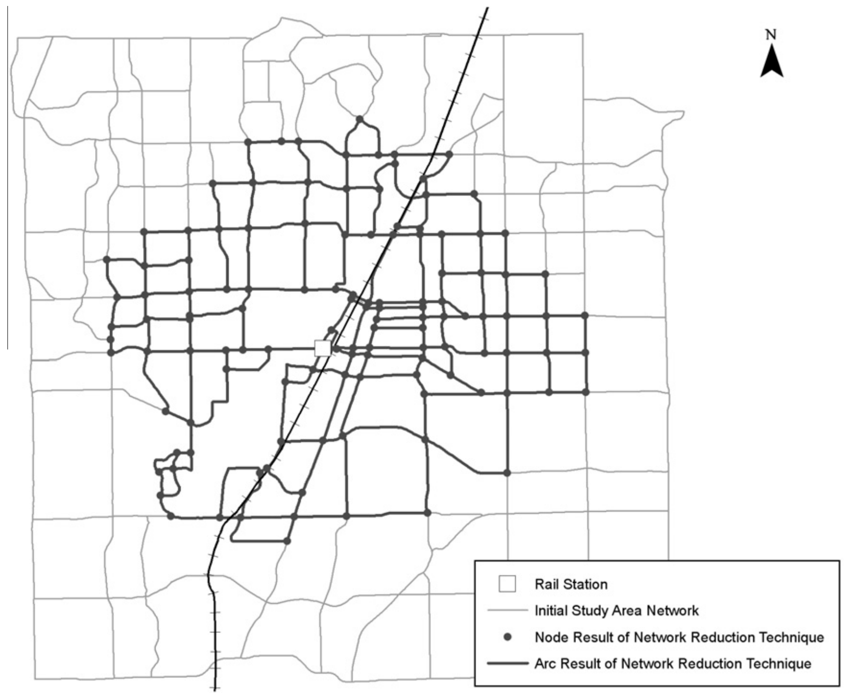

Fig. 1. Richardson, TX major road transit network.

network-reduction step is not absolutely necessary for the determination of global optimal solutions. While this step reduces the size of the problem, likely leading to faster solution times, the optimal solution will be reached (given sufficient time and memory) by the subsequent solution steps, whether or not the network is pruned.

The second stage in solving the TRANSMax model involves determining the range of possible values for the number of arcs in the optimal route. In contrast to the TSP, for the transit routing problem the number of stops (or cities) is not known in advance. Therefore, the number of links in the optimal route (the value of R) is also unknown. This has both positive and negative consequences. The most significant negative consequence is that the problem must be solved multiple times, across a range of values for R, in order to find the global optimal solution. Conversely, as will be shown, these multiple problem instances provide a set of high-quality alternate routes for decision-makers to evaluate.

Consider again the loop route example discussed above, where no overlaps or crossings may occur. Under these conditions, by observation, the lower bound of the possible range for R when determining a loop on a network is three. Considering the highly combinatorially complex nature of the route generation problem, it is desirable to have as tight as possible an upper bound on the value of R. The upper bound on the value of R can be determined by the following procedure:

(1) Order the arcs in the network by length (or other cost to traverse) from smallest to largest.
(2) Calculating a running total of length from that ordered list.
(3) When that total comes as close as possible to the distance constraint value (D) for the route without exceeding it, stop.
(4) The number of arcs that contribute to that sum is the upper bound on the value of R.

This bound is based on the fact that if, in the unlikely event that the shortest (least cost) arcs in the network could be used to create the optimal route, then you would have a route that also used the largest possible number of arcs. Using any arc not in that set (all of which are longer than all of the arcs in that set) to form a better route would require removing at least two arcs from that set in order to avoid violating the distance constraint. Therefore, R can only go down from that value with any replacement. Although it is likely that these smallest arcs do not form a valid route in terms of the other constraints (particularly the contiguity constraints), the exhaustive search procedure requires that each value in this range of R be evaluated.

One value in this range of R will provide the largest objective function value. Although it may be unintuitive, the largest value of R that results in a feasible solution will not necessarily provide the global optimal solution. Optimal solutions can be found for instances with larger values of R, however, these optimal routes may need to employ arcs with smaller service values in order to meet all of the contiguity and looping constraints. Therefore, a route with a smaller number of arcs (R) may result in a higher total service value. This occurs in the example solution provided below.

The third step in the TRANSMax solution procedure is to solve optimally for each value of R. The TRANSMax model can be solved optimally using a combination of the Simplex method (Dantzig, 1957) and a branch and bound technique to determine the integer optimal solution. In this research ILOG CPlex version 8.1 was employed to implement these procedures. The results of this three stage process are outlined below.

5. Results

5.1. Data

The model described above was tested through an application to the actual street network for a portion of Richardson, TX (Fig. 1). This area is defined by a three-mile radius around a Dallas Area Rapid Transit (DART) light rail station, which represents a waypoint for several bus routes and a stop for the light rail system. For the purposes of analysis, only those streets in this service area that could support bus traffic (given the available street attributes

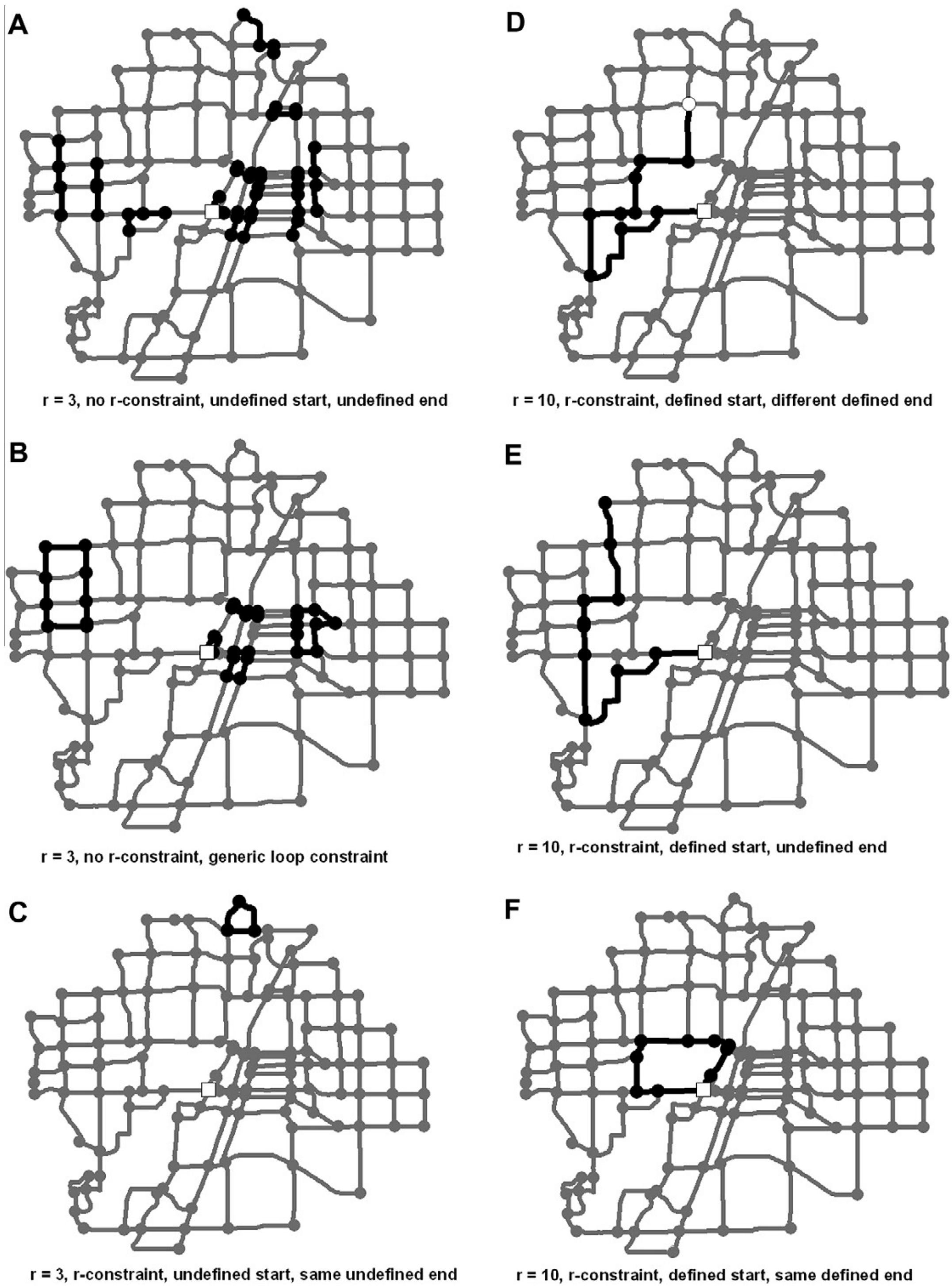

Fig. 2. TRANSMax routes with varying constraint sets.

and traffic control data) were selected. These streets have sufficient width to accommodate two-way traffic in addition to parked cars. The Geographic Information Systems (GIS) department of the city of Richardson employs a dual carriageway network representation for some of their arterial streets (Curtin et al., 2007). Since this representation did not add to the analysis of optimal routes and in fact added nodes and arcs that would complicate the solution procedure, carriageways were aggregated into a single centerline representation.

Since the objective of the TRANSMax problem is to maximize service values on arcs and nodes, service values were assigned to the network. Arc service values were assigned based on a function of the number of potential transit stops along the arc. This does demand acceptance of the assumption that more potential stops along an arc equates to more potential demand that can be served. Node service values were assigned with a random number generator since actual service values were not available at the time of this research. While the authors recognize that this is far from the ideal dataset to employ, the goal of this article is demonstrate the formulation of the TRANSMax model and the usefulness of both the model itself and its solution procedure. The service values are constants for any instance of the problem, and therefore have no effect on the combinatorial complexity of the problem or the implementation of the solution procedure. That said, future research will employ a means of determining the population with access to the transit route as a measure of service value (Biba et al., 2010).

Distance values were computed between all adjacent nodes. A user of the TRANSMax model could choose a maximum route distance value (*D*) based on a number of factors or combination of factors, including—but not limited to—the maximum time that passengers would accept for a complete route, the cost of operating the vehicle by distance, or the range of the vehicle. Again, this value is a constant in the model that must be supplied by the user, and has no influence on the complexity or solution time for the problem. For the example presented here, the maximum route distance value (*D*) was determined based on an estimation of the average speed of buses (10 miles/hour) and a maximum route travel time of 50 minutes. The 50 minute limit is a common maximum used in the DART system. This limit allows bus drivers a recovery period between trips on routes with a 1 hour cycle time. Recovery periods (or rest breaks) for drivers are known to reduce driver stress and fatigue leading to fewer accidents (Tse et al., 2006; Greiner et al., 1998). Given these constrains, the value of *D* in this instance of the TRANSMax model is 13.4 km (8.33 miles).

5.2. Computational experience

Figs. 4–6 show the results of solving the TRANSMax model for every feasible value of *R*, with constraints that require a circulator route centered on a train station. Most importantly, these results demonstrate that an integer programming approach to transit route determination is viable. Guaranteed optimal solutions can be determined for an area that comprises a normal transit route service area, without resorting to heuristic or other approximate methods. The solution time for all feasible values of *R* was roughly one day of computer time (Fig. 3). In the context of determining new, single, fixed routes for a transit organization this solution time is more than reasonable. Since transit routes are not generally redesigned on a weekly, monthly, or even yearly basis, a single day of solution time is more than sufficient to support the planning process.

Moreover, these results provide a wealth of alternatives to decision makers for evaluation and implementation. Note that in Fig. 4 there are 10 solutions that have an objective function value that is within 10% of the optimal objective function value. This means that there are 10 different "good" routes that could be implemented by a transit agency.

For the station loop instance solved here, the largest possible value of *R* is 49, although the results in the following section demonstrate that there is no feasible solution with an *R* value greater than 35.

It should also be noted that the route distance is not necessarily a good proxy for the route objective function value. In our results 18 of the 31 optimal solutions for different values of *R*, have distance values within 4% of the maximum distance limit of 13.4 km (Fig. 5). These same solutions have objective function values that range from roughly 2200 to 3250. This suggests that heuristic or manual methods that attempt to determine the optimal solution through maximizing the distance of the route can lead to significantly sub-optimal solutions.

Additionally, these alternate routes can vary substantially in terms of their route geometries. Fig. 6 shows the optimal routes where $R = 28$ and $R = 29$. These routes have very similar distance and objective function values, but they have drastically different geometries, they share few of the same network links, and they cover very different sectors of the route service area.

Depending on the distribution of service values across the network, an increase in *R* does not necessarily mean that there is an accompanying increase in total service value, or an increase in route distance. The solution procedure outlined above will find the optimal solution for every value of *R*, but as *R* increases, the route that is chosen may need to include arcs that don't have high service values in order to meet all of the model constraints. An example is shown in Fig. 7. These results were obtained using the same network as in the results presented above, with an alternate set of arc and node service values. In this instance the service value is highest with an *R* value ($R = 27$) less than the maximum feasible *R* value. In fact, the objective function value when $R = 35$ is only roughly 25% of optimal. This demonstrates that the model *must* be solved for all feasible values of *R*. It cannot be assumed that the maximum value of *R* will generate the maximum service value.

Additional results for the TRANSMax model were obtained using the variant constraint sets described above. Although detailed results are not presented here, an interesting pattern of solution difficulty presented itself and deserves note. Contrary to

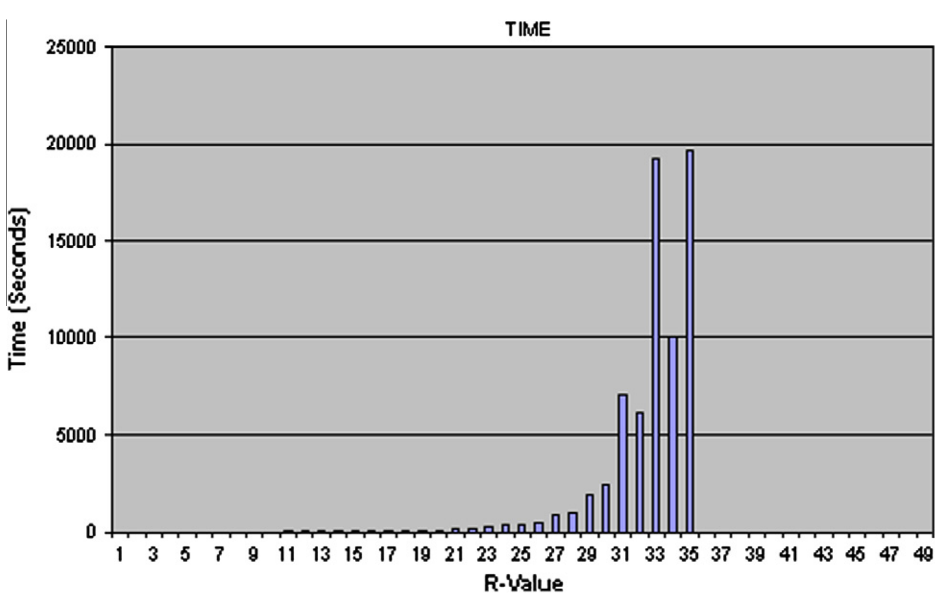

Fig. 3. Solution times for the feasible range of *R* values.

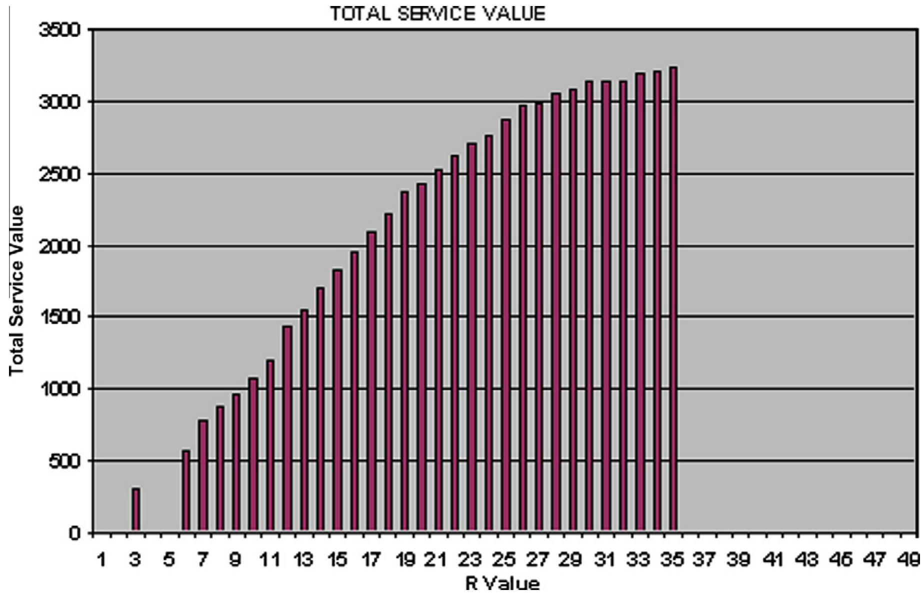

Fig. 4. Objective function values for the feasible range of *R* values.

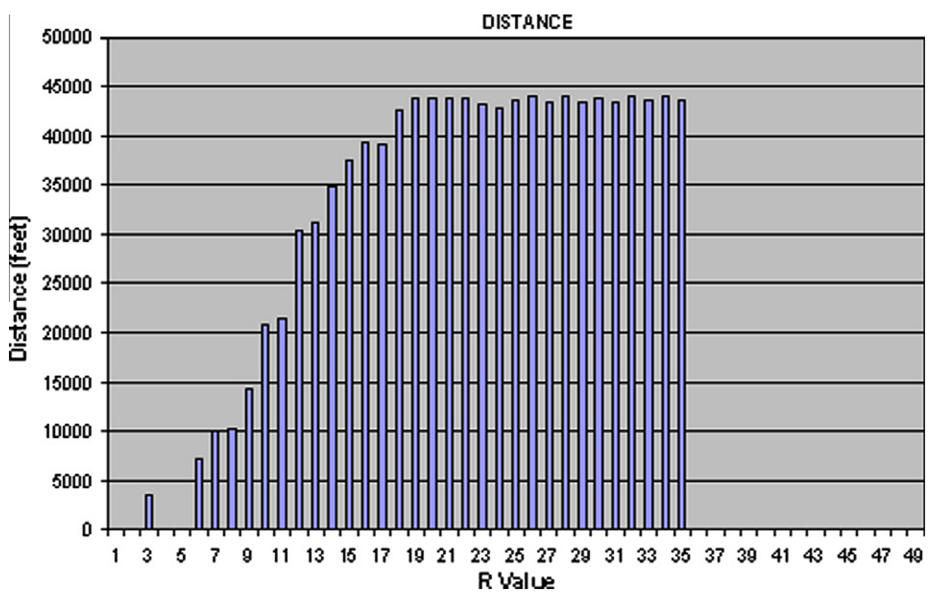

Fig. 5. Total route distances for the feasible range of *R* values.

expectations, the TRANSMax model was more difficult to solve in terms of processing time and computer memory in instances with fewer constraints. This is consistent with the findings of Magnanti and Wong (1984) who describe the solution benefits of more constrained models that provide a tighter bound to the linear programming relaxation of the model, and which have a richer collection of linear programming dual variables.

6. Conclusions and future research

In this research a new method for determining optimal transit routes has been presented. A formulation for the TRANSMax model, and variant constraint sets that can be used to generate different route types (feeder, circulators, etc.) were given. This model maximizes the overall service value of a route rather than attempting to minimize cost. In the TRANSMax model cost is represented by route distance, and this distance acts as a constraint on the extent of the route. The mathematical formulation borrows from the structure of linear programming problems designed for the traveling salesman problem. A three-step solution procedure was developed, including (1) a network reduction heuristic, (2) the determination of the range of possible values for the number of arcs in the optimal route, and (3) the solution of all instances of the TRANSMax model using that range of values to exhaustively determine the global optimal solution. This method provides a range of high-quality alternate routes for consideration in the decision making process.

The TRANSMax model was tested on an existing street network. This test demonstrated that the optimal solution was not necessarily found with the largest feasible value of *R*, that distance was not a good determinant of objection function value, and that alternate route geometries may cover substantially different sectors of the study area while having very similar lengths and service values.

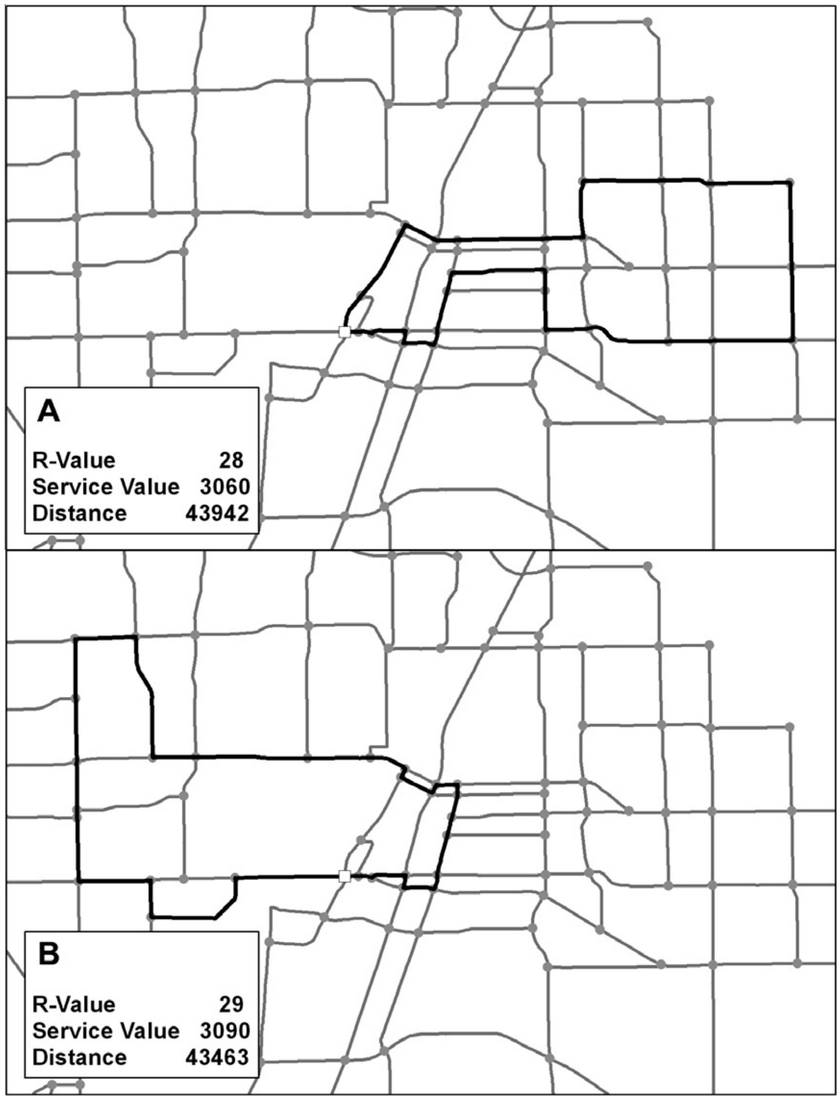

Fig. 6. Two near-optimal solutions with contrasting route geometries.

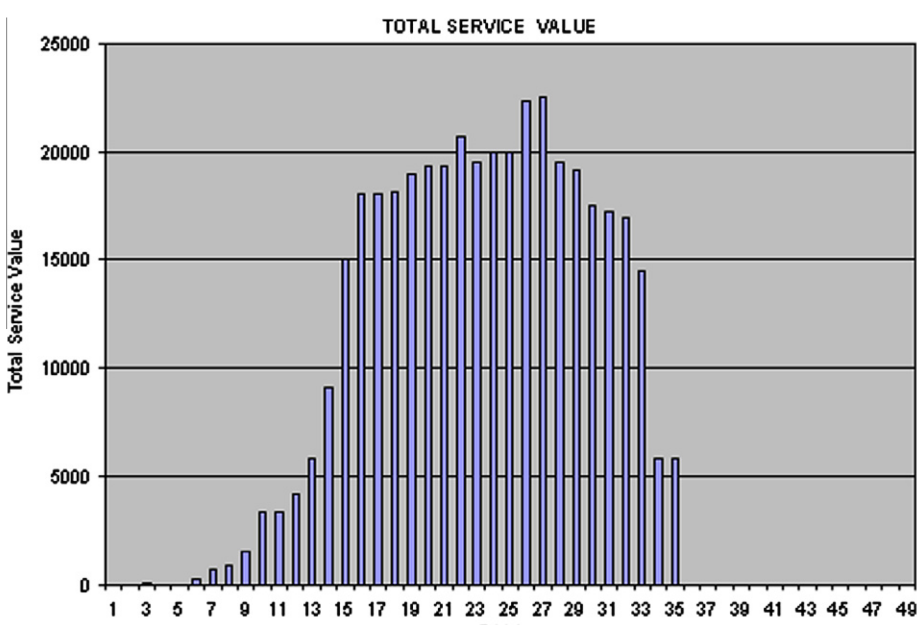

Fig. 7. Objective function values for an alternate dataset.

Most importantly, however, this test confirmed that an optimal integer programming approach to transit route determination is viable.

There are many avenues for future research with the TRANSMax model as a basis. The authors feel that the most dramatic improvement to the method would result from tighter bounds on the range of values for *R*. Although our results demonstrate that instances of the TRANSMax model with all feasible values of *R* can be solved in a reasonable amount of time, it would be preferable to solve even fewer. Similarly, the network reduction element of the solution procedure could be refined for variant formulations, resulting in smaller problem instances.

Additional research is needed regarding the best way to determine service values on nodes and arcs. There are many possible values that are functions of population, employment, commercial square footage, or other measures of attraction to destinations on the network. It may be valuable to differentiate between source and destination service values, and then balance those service values on any given route. Additionally, sensitivity analyses could be performed to determine which of the alternate acceptable routes is more robust under changes in the estimates of service values. Lastly, the authors envision additional formulations that would model the capacity of the transit vehicles being routed (Zachariadis et al., 2009), the frequencies of service across the routes (Zhao and Zeng, 2008), and the simultaneous location of facilities (e.g. depots, bus stops) alongside optimal routing (Nagy and Salhi, 2007).

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